LEARNING OUTCOMES

1. Formulate Alternatives
2. PW of equal-life alternatives
3. PW of different-life alternatives
4. Future Worth analysis
5. Capitalized Cost analysis
Formulating Alternatives

Two types of economic proposals

**Mutually Exclusive (ME) Alternatives:** *Only one* can be selected; Compete against each other

**Independent Projects:** *More than one* can be selected; Compete only against DN

Do Nothing (DN) – An ME alternative or independent project to maintain the current approach; no new costs, revenues or savings
Formulating Alternatives

Two types of cash flow estimates

- **Revenue**: Alternatives include estimates of costs (cash outflows) and revenues (cash inflows)

- **Cost**: Alternatives include only costs; revenues and savings assumed equal for all alternatives; also called *service alternatives*
PW Analysis of Alternatives

- Convert all cash flows to PW using MARR
- Precede costs by minus sign; receipts by plus sign

**EVALUATION**

- For one project, if PW > 0, it is justified
- For mutually exclusive alternatives, select one with numerically largest PW
- For independent projects, select all with PW > 0
Selection of Alternatives by PW

For the alternatives shown below, which should be selected if they are (a) mutually exclusive; (b) independent?

<table>
<thead>
<tr>
<th>Project ID</th>
<th>Present Worth</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$30,000</td>
</tr>
<tr>
<td>B</td>
<td>$12,500</td>
</tr>
<tr>
<td>C</td>
<td>$-4,000</td>
</tr>
<tr>
<td>D</td>
<td>$2,000</td>
</tr>
</tbody>
</table>

Solution:
(a) Select numerically largest PW; alternative A
(b) Select all with PW > 0; projects A, B & D
Example: PW Evaluation of Equal-Life ME Alts.

Alternative X has a first cost of $20,000, an operating cost of $9,000 per year, and a $5,000 salvage value after 5 years. Alternative Y will cost $35,000 with an operating cost of $4,000 per year and a salvage value of $7,000 after 5 years. At an MARR of 12% per year, which should be selected?

Solution: Find PW at MARR and select numerically larger PW value

$$PW_X = -20,000 - 9000(P/A, 12\%, 5) + 5000(P/F, 12\%, 5) = -$49,606$$

$$PW_Y = -35,000 - 4000(P/A, 12\%, 5) + 7000(P/F, 12\%, 5) = -$45,447$$

Select alternative Y
PW of Different-Life Alternatives

**Must** compare alternatives for *equal service*
(i.e., alternatives must *end* at the same time)

Two ways to compare equal service:

- Least common multiple (LCM) of lives
- Specified study period

(The LCM procedure is used unless otherwise specified)
Assumptions of LCM approach

- Service provided is needed over the LCM or more years

- Selected alternative can be repeated over each life cycle of LCM in exactly the same manner

- Cash flow estimates are the same for each life cycle (i.e., change in exact accord with the inflation or deflation rate)
Example: Different-Life Alternatives

Compare the machines below using present worth analysis at $i = 10\%$ per year

<table>
<thead>
<tr>
<th></th>
<th>Machine A</th>
<th>Machine B</th>
</tr>
</thead>
<tbody>
<tr>
<td>First cost, $</td>
<td>20,000</td>
<td>30,000</td>
</tr>
<tr>
<td>Annual cost, $/year</td>
<td>9000</td>
<td>7000</td>
</tr>
<tr>
<td>Salvage value, $</td>
<td>4000</td>
<td>6000</td>
</tr>
<tr>
<td>Life, years</td>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

**Solution:** LCM = 6 years; repurchase A after 3 years

$$PW_A = -20,000 - 9000(P/A,10\%,6) - 16,000(P/F,10\%,3) + 4000(P/F,10\%,6)$$
$$PW_A = -$68,961$$

$$PW_B = -30,000 - 7000(P/A,10\%,6) + 6000(P/F,10\%,6)$$
$$PW_B = -$57,100$$

Select alternative B
PW Evaluation Using a Study Period

- Once a study period is specified, all cash flows after this time are ignored.

- Salvage value is the estimated market value at the end of study period.

Short study periods are often defined by management when business goals are short-term.

Study periods are commonly used in equipment replacement analysis.
Example: Study Period PW Evaluation

Compare the alternatives below using present worth analysis at i = 10% per year and a 3-year study period

<table>
<thead>
<tr>
<th></th>
<th>Machine A</th>
<th>Machine B</th>
</tr>
</thead>
<tbody>
<tr>
<td>First cost, $</td>
<td>-20,000</td>
<td>-30,000</td>
</tr>
<tr>
<td>Annual cost, $/year</td>
<td>-9,000</td>
<td>-7,000</td>
</tr>
<tr>
<td>Salvage/market value, $</td>
<td>4,000</td>
<td>6,000 (after 6 years)</td>
</tr>
<tr>
<td>Life, years</td>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

Solution: Study period = 3 years; disregard all estimates after 3 years

\[
PW_A = -20,000 - 9000(P/A,10\%,3) + 4000(P/F,10\%,3) = -$39,376
\]

\[
PW_B = -30,000 - 7000(P/A,10\%,3) + 10,000(P/F,10\%,3) = -$39,895
\]

Marginally, select A; different selection than for LCM = 6 years

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Future Worth Analysis

**Must** compare alternatives for *equal service* (i.e. alternatives must end at the same time)

Two ways to compare equal service:

- Least common multiple (LCM) of lives
- Specified study period

(The LCM procedure is used unless otherwise specified)
**FW of Different-Life Alternatives**

Compare the machines below using future worth analysis at \( i = 10\% \) per year

<table>
<thead>
<tr>
<th></th>
<th>Machine A</th>
<th>Machine B</th>
</tr>
</thead>
<tbody>
<tr>
<td>First cost, $</td>
<td>-20,000</td>
<td>-30,000</td>
</tr>
<tr>
<td>Annual cost, $/year</td>
<td>-9000</td>
<td>-7000</td>
</tr>
<tr>
<td>Salvage value, $</td>
<td>4000</td>
<td>6000</td>
</tr>
<tr>
<td>Life, years</td>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

**Solution:** \( \text{LCM} = 6 \) years; repurchase A after 3 years

\[
\begin{align*}
\text{FW}_A &= -20,000(F/P,10\%,6) - 9000(F/A,10\%,6) - 16,000(F/P,10\%,3) + 4000 \\
&= -$122,168 \\
\text{FW}_B &= -30,000(F/P,10\%.6) - 7000(F/A,10\%,6) + 6000 \\
&= -$101,157
\end{align*}
\]

**Select B** (Note: PW and FW methods will *always* result in *same selection*)
1. Machines that have the following costs are under consideration for a robotized welding process. Using an interest rate of 10% per year, determine which alternative should be selected on the basis of a present worth analysis.

<table>
<thead>
<tr>
<th></th>
<th>Machine X</th>
<th>Machine Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>First cost, $</td>
<td>-250,000</td>
<td>-430,000</td>
</tr>
<tr>
<td>Annual OC, $ per yr</td>
<td>-60,000</td>
<td>-40,000</td>
</tr>
<tr>
<td>Salvage value, $</td>
<td>70,000</td>
<td>95,000</td>
</tr>
<tr>
<td>Life, years</td>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>
2. Compare the alternatives shown below on the basis of a future worth analysis, using an rate of 8% per year.

<table>
<thead>
<tr>
<th></th>
<th>P</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>First cost, $</td>
<td>-23,000</td>
<td>-30,000</td>
</tr>
<tr>
<td>Annual OC, $ per year</td>
<td>-4,000</td>
<td>-2,500</td>
</tr>
<tr>
<td>Salvage value, $</td>
<td>3,000</td>
<td>1,000</td>
</tr>
<tr>
<td>Life, years</td>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>
Capitalized Cost (CC) Analysis

CC refers to the present worth of a project with a very long life, that is, PW as n becomes *infinite*

Basic equation is: \[ CC = P = \frac{A}{i} \]

“\( A \)” essentially represents the *interest* on a perpetual investment

For example, in order to be able to withdraw $50,000 per year forever at \( i = 10\% \) per year, the amount of capital required is \( 50,000/0.10 = $500,000 \)

For *finite life* alternatives, convert all cash flows into an \( A \) value over *one life cycle* and then divide by \( i \)
Example: Capitalized Cost

Compare the machines shown below on the basis of their capitalized cost. Use $i = 10\%$ per year

<table>
<thead>
<tr>
<th></th>
<th>Machine 1</th>
<th>Machine 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>First cost, $</td>
<td>-20,000</td>
<td>-100,000</td>
</tr>
<tr>
<td>Annual cost, $/year</td>
<td>-9000</td>
<td>-7000</td>
</tr>
<tr>
<td>Salvage value, $</td>
<td>4000</td>
<td>-----</td>
</tr>
<tr>
<td>Life, years</td>
<td>3</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>

**Solution:** Convert machine 1 cash flows into A and then divide by $i$

\[
A_1 = -20,000(A/P,10\%,3) - 9000 + 4000(A/F,10\%,3) = -$15,834
\]

\[
CC_1 = -15,834 / 0.10 = -$158,340
\]

\[
CC_2 = -100,000 - 7000/ 0.10 = -$170,000
\]

Select machine 1
Summary of Important Points

- PW method converts all cash flows to present value at MARR
- Alternatives can be mutually exclusive or independent
- Cash flow estimates can be for revenue or cost alternatives
- PW comparison must always be made for equal service
- Equal service is achieved by using LCM or study period
- Capitalized cost is PW of project with infinite life; \( CC = P = \frac{A}{i} \)